

Sign and narrative restrictions

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Worked calculations for orthogonal rotations, joint sign tests, incoherent model averages, and dated shock restrictions.

Based on the 2024 BSE course taught by Luca Gambetti and Konstantin Boss. I have revised the classroom code and added the accompanying explanations. The notes identify the papers, data, and assumptions used in each application. These are my course notes, not an official BSE publication. A classroom replication should not be read as a replication of every result in its underlying paper.

1 Orthogonal rotations

First, $QQ' = I_2$. The impact matrix is

$$B = \begin{pmatrix} 0 & -2 \\ 1 & -1 \end{pmatrix}, \quad BB' = \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix} = SS'.$$

For example, the covariance between the two reduced-form innovations is $0 \cdot 1 + (-2)(-1) = 2$, exactly as with S . Both factorizations therefore fit the same innovation covariance. Neither calculation labels a monetary, supply, or demand shock. Economic restrictions must give those columns an interpretation. Checking $BB' = SS'$ verifies observational equivalence, not causal identification.

2 A joint sign test

Multiply the first row by $+1$ and the second by -1 . Every resulting entry of R_1 is positive, so its orientation is accepted. Every active sign of R_2 is reversed; multiplying the entire shock column by -1 returns R_1 and preserves orthogonality.

For R_3 , the impact signs pass and the horizon-one signs fail. Reversing the column would make horizon one pass but impact fail. The system is rejected. A horizon-specific reversal would replace a single structural shock with differently oriented objects at different horizons; it cannot be written as $C_h S q_j$ for one common q_j .

The direct code check is:

```
% Apply one supply-sign vector jointly to both horizons.
direction = tsma.ident.sign_orientation([1 -2; -1 3], [1; -1]);
% Zero denotes rejection, not an unrestricted or zero-valued shock.
assert(direction == 0);
```

3 Combining models

The average is $\bar{B} = \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}$. Its columns remain perpendicular, but each has squared length 1/2. Consequently,

$$\bar{B}\bar{B}' = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \neq I_2.$$

Each original model reproduces the data covariance; their entrywise average does not. With two observations, an interpolated sample median can equal this average as well. More generally, selecting marginal quantiles separately across variables and horizons does not enforce the nonlinear restrictions tying a complete model together. A plotted summary can be useful descriptively, but it is not automatically a structural impact matrix. Use a saved draw for a decomposition and verify its covariance identity before interpreting the result.

4 A dated restriction

The first equation gives $\varepsilon_{1,t^*} = 1$. The second gives $1 + 2\varepsilon_{2,t^*} = -1$, hence $\varepsilon_{2,t^*} = -1$. The proposed positive-second-shock restriction therefore rejects this model.

Reversing the second column produces $\tilde{B} = \begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$ and recovers $\tilde{\varepsilon}_{2,t^*} = +1$. The fitted reduced-form innovation and its covariance are unchanged. But all impulse responses to that second shock have reversed too. Its economic label and any response-sign restrictions must be checked in the same orientation. A narrative sign cannot be made meaningful by reversing the shock after observing whether the desired historical claim passes.

For an empirical date, match its calendar code to `model.periods` before recovering shocks. If that date is not in the residual sample, the exercise has no recovered innovation at that date; inventing a zero would add a different assumption.