

Impulse responses and decompositions

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Worked calculations for horizon indexing, variance shares, historical reconstruction, and the interpretation of pointwise bootstrap intervals.

Based on the 2024 BSE course taught by Luca Gambetti and Konstantin Boss. I have revised the classroom code and added the accompanying explanations. The notes identify the papers, data, and assumptions used in each application. These are my course notes, not an official BSE publication. A classroom replication should not be read as a replication of every result in its underlying paper.

1 Impact and one-step uncertainty

Impact is $\Theta_0 = B$. One period later,

$$\Theta_1 = AB = \begin{pmatrix} .5 & 0 \\ .2 & .4 \end{pmatrix}.$$

For variable two, shock one contributes $1^2 + .2^2 = 1.04$ to the two-step forecast error variance. Shock two contributes $2^2 + .4^2 = 4.16$. The total is 5.2, so the shares are $1.04/5.2 = .2$ and $4.16/5.2 = .8$.

At forecast date t , the error for y_{t+2} contains $B\varepsilon_{t+2} + AB\varepsilon_{t+1}$. An innovation dated t is already in the conditioning information and is not part of the forecast error. This is why $\Theta_2 = A^2B$ does not enter. The code's second FEVD slice therefore uses the first two IRF slices, representing economic response horizons zero and one.

2 Reconstructing a history

The realized path is

$$y_1 = 1 + .5(2) + 1 = 3, \quad y_2 = 1 + .5(3) - 2 = .5, \quad y_3 = 1 + .5(.5) + 0 = 1.25.$$

With the same initial condition and intercept but zero later innovations, the baseline remains $a_1 = a_2 = a_3 = 2$. The summed shock contribution is

$$d_1 = 1, \quad d_2 = .5(1) - 2 = -1.5, \quad d_3 = .25(1) + .5(-2) + 0 = -.75.$$

Adding baseline and shocks gives 3, .5, and 1.25, exactly the observed path. At date three, the initial positive shock still contributes .25 and the negative second shock contributes -1. The zero third innovation has no direct contribution but does not erase the effects of previous shocks.

If only d_t were plotted and labeled as the level of y_t , every observation would be understated by two. Demeaning may simplify a decomposition in some settings, but it cannot justify silently discarding the initial-state path.

3 Reading a bootstrap interval

The lower endpoint uses index $\lceil 500(.16) \rceil = 80$ and the upper endpoint uses index $\lceil 500(.84) \rceil = 420$. The algorithm uses these observed order statistics; it does not interpolate between adjacent draws. Another percentile convention would differ slightly in a finite bootstrap sample and should be stated explicitly.

Under the exercise's artificial independence assumption, the probability both intervals cover is $.68^2 = .4624$, not $.68$. For an actual VAR, responses at nearby horizons share coefficients and bootstrap innovations, so their coverage events are dependent. Multiplying marginal coverage probabilities is therefore not the correct simultaneous-coverage calculation.

A simultaneous band needs a joint calibration, for example to a suitable maximum statistic over the selected horizon set. Merely changing the plotted label from pointwise to simultaneous cannot provide that calibration. Also distinguish two uncertainties: more bootstrap draws reduce numerical variation in the estimated quantiles, whereas more observed data may reduce statistical uncertainty about the underlying model.

4 Growth and levels

For growth, the shock-specific two-step variances are

$$V_1^g = 1^2 + (-1)^2 = 2, \quad V_2^g = 1^2 + 1^2 = 2.$$

Each shock therefore has share one half. For the level, cumulate each shock's growth responses first. The response sequences become $(1, 0)$ for shock one and $(1, 2)$ for shock two. Hence

$$V_1^\ell = 1^2 + 0^2 = 1, \quad V_2^\ell = 1^2 + 2^2 = 5.$$

The level shares are $1/6$ and $5/6$. Shock one's second growth response reverses its first level effect, while shock two's growth responses reinforce one another. The difference is lost if the growth responses are squared before cumulation. Both sets of shares add to one, so an adding-up check alone would not reveal that the wrong outcome had been decomposed.